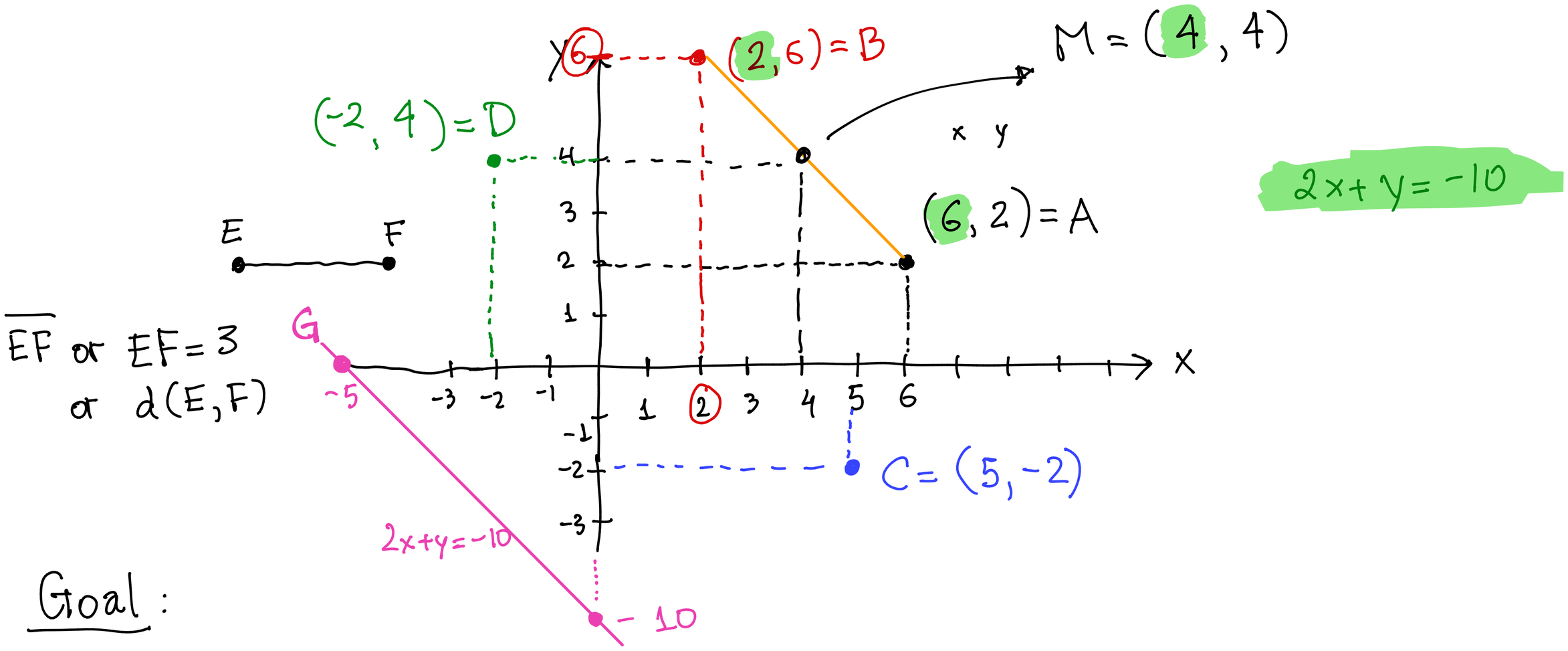
Class 5 - Rectangular Coordinate System



Goal:

1. Midpoint of a line segment (Without looking at the graph)
2. Length of a line segment
3. Find x-intercepts & y-intercepts

Review

Operations with fractions

Ex 1: Compute $\frac{1}{10} - \frac{6}{35}$.

10, 35	2
5, 35	5
1, 7	7
1, 1	

LCD = 70

$$\frac{7\cdot 1}{7\cdot 10} - \frac{6\cdot 2}{35\cdot 2} = \frac{7}{70} - \frac{12}{70}$$

$$= \frac{7-12}{70} = -\frac{5}{70} \overset{\div 5}{\underset{\div 5}{}} = -\frac{1}{14}.$$

On the calculator: (1 / 10) - (6 / 35) = ANSWER

Convert to fraction: 2nd → PRB → (2nd → $A^{b/c}$) if ANSWER > 1

1.5 , $1\frac{1}{2}$, $\frac{3}{2}$ → lowest terms

Average of two numbers

Ex 2. Average of A & B $= \frac{A+B}{2}$.

Compute the average of $\frac{1}{4}$ and $\frac{2}{7}$.

$$\frac{\frac{7\cdot 1}{7\cdot 4} + \frac{2\cdot 4}{7\cdot 4}}{2} = \frac{\frac{7}{28} + \frac{8}{28}}{2} = \frac{\frac{15}{28}}{\frac{2}{1}} = \frac{15}{28}\cdot\frac{1}{2} = \frac{15}{56}$$

On calculator: (1/4 + 2/7) ÷ 2 → 2nd, PRB → 15/56.

Simplify Roots

Ex 3: Use the property $\sqrt{a\cdot b} = \sqrt{a}\cdot\sqrt{b}$ to simplify $\sqrt{1584}$.

→ Prime numbers 2, 3, 5, 7, 11, ...

→ $2^2 = 4$, $3^2 = 9$, $5^2 = 25$, $7^2 = 49$

$$\sqrt{1584} = \sqrt{4\cdot 396} = \sqrt{4}\cdot\sqrt{396} = 2\sqrt{396}$$

$$= 2\sqrt{4\cdot 99} = 2\sqrt{4}\cdot\sqrt{99} = 2\cdot 2\cdot\sqrt{99}$$

$$= 4\sqrt{9\cdot 11} = 4\cdot\sqrt{9}\cdot\sqrt{11} = 4\cdot 3\sqrt{11}$$

$$= 12\sqrt{11}.$$

Solve linear Equations

Ex 4: Solve the following linear equations:

a) $\frac{y+2}{5} = 8$ ·5 → $y + 2 = 40$ −2 −2

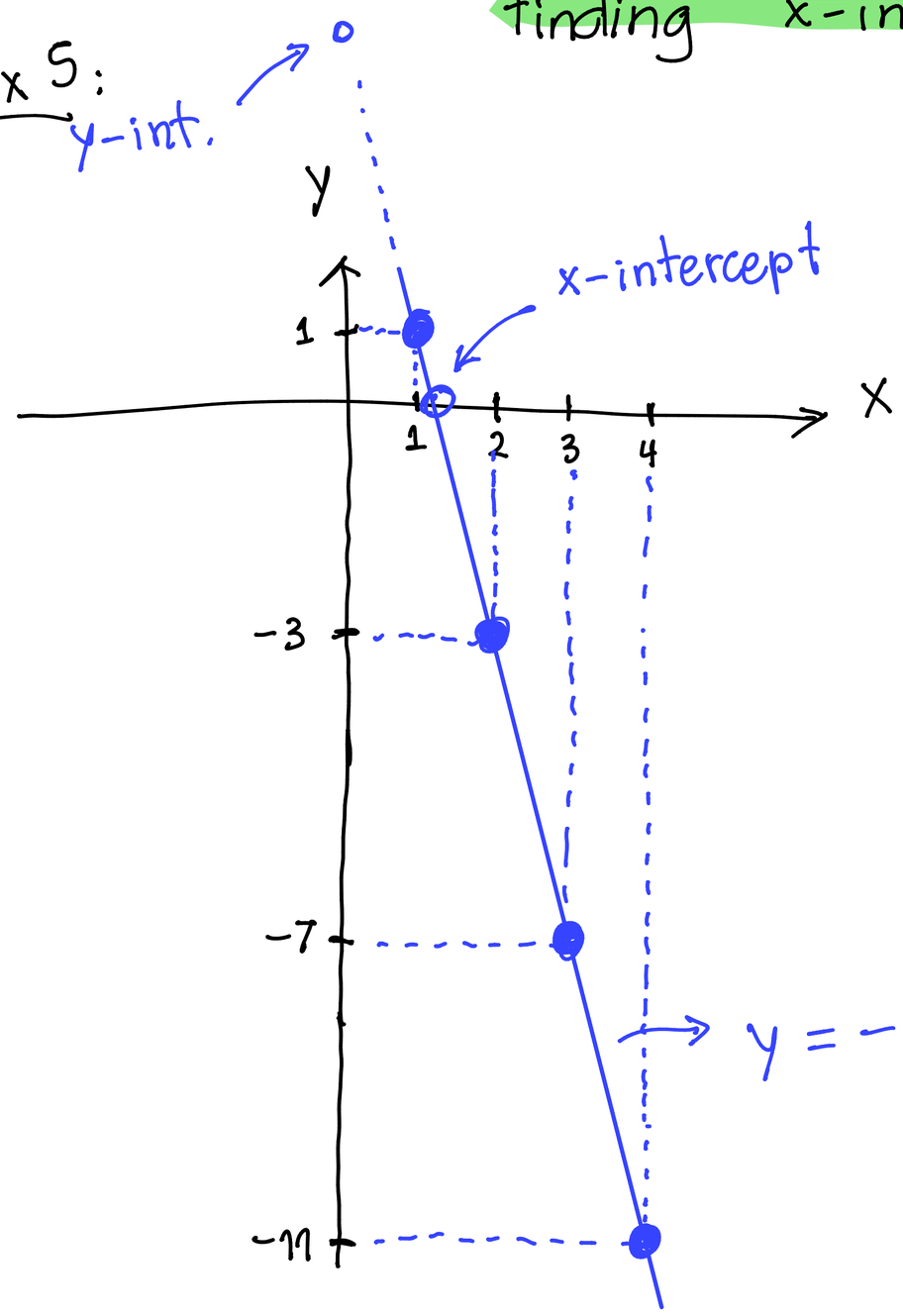
$\boxed{y = 38}$

b) $\frac{(x+8)}{8}$ ·24 $= \frac{(5)}{6}$ ·24 → $3x + 24 = 20$ −24 −24

$\frac{3x}{3} = \frac{-4}{3}$ → $\boxed{x = -4/3}$

Finding x-int & y-ints

Ex 5:



$y = -4x + 5$

x	$y = -4x + 5$	
1	$y = -4 \cdot 1 + 5 = 1$	$\rightarrow (1, 1)$
2	$y = -4 \cdot 2 + 5 = -3$	$\rightarrow (2, -3)$
3	$y = -4 \cdot 3 + 5 = -7$	$\rightarrow (3, -7)$
4	$y = -4 \cdot 4 + 5 = -11$	$\rightarrow (4, -11)$

x-intercept : set $y = 0$

$y = -4x + 5 \Rightarrow 0 = -4x + 5 \Rightarrow$

$+4x \quad +4x$

$\frac{4x}{4} = \frac{5}{4} \Rightarrow \boxed{x = \frac{5}{4}}$ or 1.25

y-intercept : set $x = 0$

$y = -4x + 5 \Rightarrow y = -4 \cdot 0 + 5 = 5$

Ex 6: Find the intercepts of $y = x^2 - x - 30$.

x-intercepts : set $y = 0$.

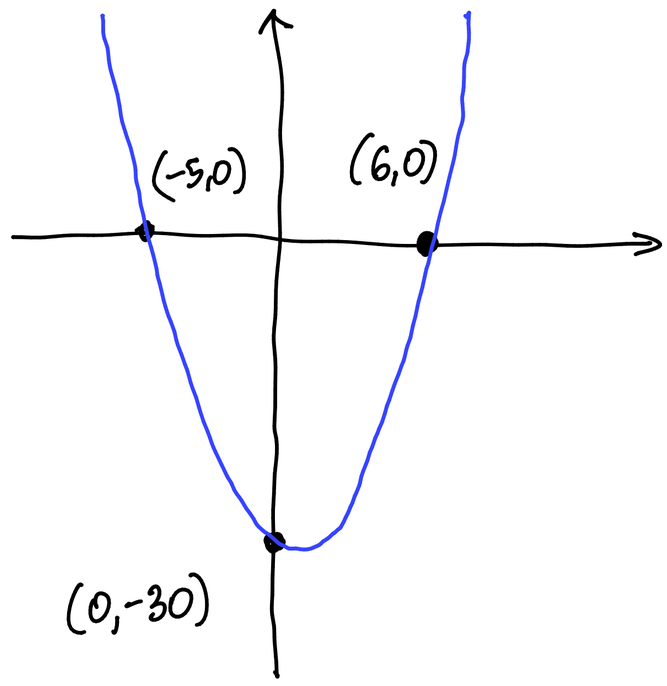
$AC = -30, B = -1 \rightarrow 5$ & -6

prod sum

$0 = x^2 - x - 30 \Rightarrow 0 = (x + 5)(x - 6)$

$a \quad b$

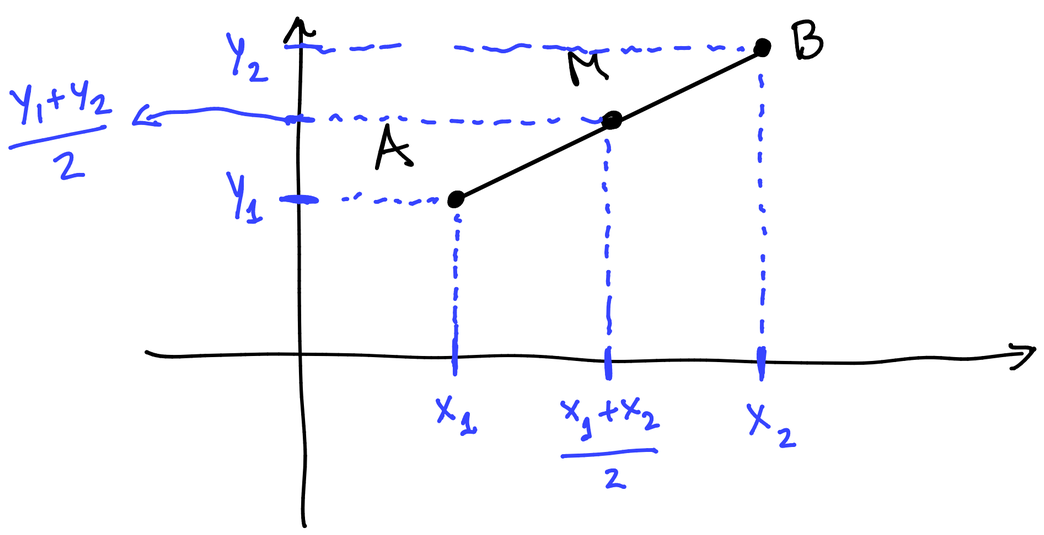
$x + 5 = 0 \Rightarrow \boxed{x = -5}$ or $x - 6 = 0 \Rightarrow \boxed{x = 6}$



y-intercepts : set $x = 0$

$y = 0^2 - 0 - 30 \Rightarrow y = -30$

Midpoint of a line segment

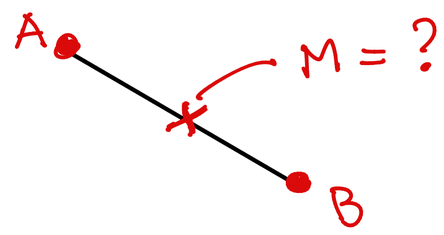


$A = (x_1, y_1)$

$B = (x_2, y_2)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Ex 7: Find the midpoint of $A=\left(\frac{7}{2},-\frac{4}{7}\right)$ & $B=\left(\frac{2}{5},\frac{3}{4}\right)$.



$$M=\left(\frac{\frac{7}{2}+\frac{2}{5}}{2},\frac{-\frac{4}{7}+\frac{3}{4}}{2}\right)=\left(\frac{\frac{35}{10}+\frac{4}{10}}{2},\frac{\frac{-16}{28}+\frac{21}{28}}{2}\right)$$

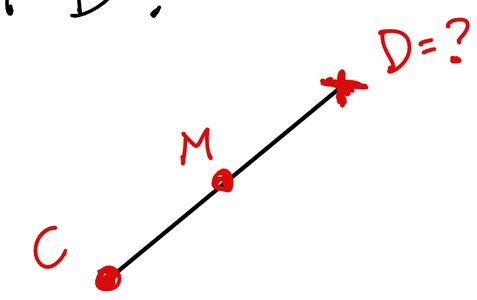
$$=\left(\frac{\frac{39}{10}}{2},\frac{\frac{5}{28}}{2}\right)=\left(\frac{39}{20},\frac{5}{56}\right).$$

Exercise: If $M=\left(\frac{1}{2},\frac{3}{5}\right)$ is the midpoint of the line segment CD and $C=\left(4,-\frac{1}{3}\right)$, what are the coordinates of point D?

$D=(x,y)$

$M=\left(\frac{1}{2},\frac{3}{5}\right)$ $\longrightarrow$ $\frac{4+x}{2}=\frac{1}{2}$ and $\frac{-\frac{1}{3}+y}{2}=\frac{3}{5}$

$C=\left(4,-\frac{1}{3}\right)$

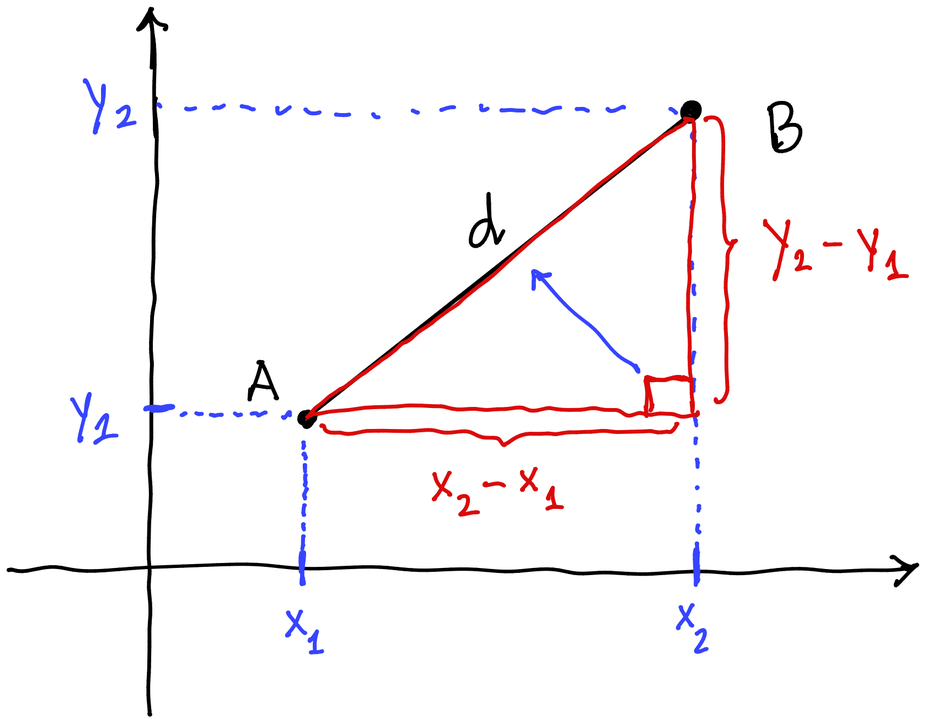


rewrite $\rightarrow$ $-\frac{1}{6}+\frac{y}{2}=\frac{3}{5}$

$\rightarrow \left(\frac{4+x}{2}\right)2=\left(\frac{1}{2}\right)2 \Rightarrow 4+x=1 \Rightarrow x=-3$ $\longrightarrow$ $D=\left(-3,\frac{23}{15}\right)$

$\rightarrow \left(-\frac{1}{6}\right)30+\left(\frac{y}{2}\right)30=\left(\frac{3}{5}\right)30 \Rightarrow -5+15y=18 \Rightarrow \frac{15y}{15}=\frac{23}{15} \Rightarrow y=\frac{23}{15}$

Distance of two points



$A=(x_1,y_1)$, $B=(x_2,y_2)$

Pythagorean Theorem

$a^2+b^2=c^2$ ← hypotenuse

$\sqrt{d^2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \Rightarrow d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Ex 8: Find the distance between $A=(4,3)$ and $B=(-1,15)$.

$\underset{x_1}{4}\ \underset{y_1}{3}$ $\underset{x_2}{-1}\ \underset{y_2}{15}$

$$d=\sqrt{(-1-4)^2+(15-3)^2}=\sqrt{(-5)^2+(12)^2}=\sqrt{25+144}=\sqrt{169}=13.$$

Ex 9: Find the distance between $A=(2,-3)$ and $B=(8,9)$.

$\underset{x_1}{2}\ \underset{y_1}{-3}$ $\underset{x_2}{8}\ \underset{y_2}{9}$

$$d=\sqrt{(8-2)^2+(9-(-3))^2}=\sqrt{6^2+12^2}=\sqrt{36+144}=\sqrt{180}.$$

~~$4=2^2$~~, $9=3^2$, $25=5^2$.

$$\sqrt{180}=\sqrt{4\cdot 45}=\sqrt{4}\cdot\sqrt{45}=2\sqrt{45}$$

$$2\sqrt{45}=2\sqrt{9\cdot 5}=2\sqrt{9}\cdot\sqrt{5}=2\cdot 3\sqrt{5}\Rightarrow \boxed{d=6\sqrt{5}}$$